Addition
Addition Vocabulary
The symbol for addition is (+)

Other words for addition are: add, adding, plus, total amount, counting on, combining, how much altogether, sum of.

Addition (+) and subtraction (-) are inverse operations. That means they go together, but they do opposite things. Addition makes numbers bigger and subtraction makes numbers smaller.

For example, if you know that 5+7=12 then you also know: 7+5=12, 12-5=7, 12-7=5. We call these number families.

Place Value & Addition: If you know that 5+7=12 then you should also know that 50+70=120, 500+700=1200 & 5000+7000=12000

MENTAL CALCULATIONS
These are a selection of mental calculation strategies:
Mental recall of number bonds/compliments to 10 (Rainbow Facts)
6 + 4 = 10  □ + 4 = 10  9 + 1 = 10  19 + □ = 20
8 + 2 = 10  7 + 3 = 10  5 + 5 = 10

Use doubles/near doubles
6+6=12  7+7=14
6 + 7 = double 6 + 1 = 13
6+3+6+8+4+8=? (8+8)+(6+6)+(3+4)=35

Addition using partitioning and recombinining
34 + 45 = (30 + 40) + (4 + 5) = 79

Split Strategy: Add the tens together, then add the ones together and then combine them:
56+23=? (50+20)+(6+3)=79

Counting on or back in repeated steps of 1, 10, 100, 1000
17+4=21  17,18,19,20,21
86 + 57 = 143 (by counting on in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)
Making Tens: If you have to add several numbers together, find ones that add to ten first:
\[ 8+7+9+2+3=? \quad (8+2)+(7+3)+9=29 \]

Bridging To A 10: breaking the second number up and adding to a ten first. 28+5=? can be bridged to (28+2=30)+3=33

Add the nearest multiple of 10, 100 and 1000 and adjust
\[ 24 + 19 = 24 + 20 - 1 = 43 \]
\[ 458 + 71 = 458 + 70 + 1 = 529 \]

Use the relationship between addition and subtraction
\[ 36 + 19 = 55 \quad 19 + 36 = 55 \]
\[ 55 - 19 = 36 \quad 55 - 36 = 19 \]

Alternative Written Methods

**Adding from left to right**

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When using this method, you add the columns from left to right until you get a “final” answer.
Add the hundreds, then the tens and finally the ones.
900 goes straight down.
70 and 40 is 110; put it down in the correct column.
6,5,8 & 3 is 22; 20 in the tens column and 2 in the ones column.
Add the hundred again, then the tens.

**Adding using Extended Notation**

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<td>9 4 3 = 900 + 40 + 3</td>
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<td>= 900 +110 +22</td>
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<td>= 900 +132</td>
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<td>=1,032</td>
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Expand the numbers as extended notation, so the child can see and understand the value of each number. The numbers can then be added in any order; however the numbers must remain in their correct columns. When there are tens in the ones column tell the child to send tens “back home” and make the adjustments accordingly. The same thing happens when there are hundreds in the thousands column, etc.
**Subtraction**

**Subtraction Vocabulary**
The symbol for subtraction is (-)

**Other words for subtraction are:** less than, counting back, minus, take away

**Alternative Written Methods**

**Adding On**

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Children can use the ‘adding on’ method to complete subtraction problems. This should involve rounding to the nearest tens or hundreds & so on.
To get to 100 from 85 is 15; from 100 to 500 is 400 so that becomes 415 plus the 13, the difference is 428.
When using this method, the children can write the number sentence horizontally, i.e. 513-85=

**Regrouping with Expansion**

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= 500 + 10 + 13

With this method you tell the child a story about the ones, tens and hundreds.
That the 10 from the tens column is going to help out the ones and turn it into a 13, then you can subtract 5 from 13.
Same goes for the hundreds, tell the child 100 is going to help out the tens and make it 10 so that 80 can be subtracted; then we are left with 400. Don’t forget to make the necessary changes as you ‘shift’

**Using Negative Subtraction**

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<tr>
<td>-</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>2</td>
<td>2</td>
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</table>

When using Negative Subtraction, the children can work from left to right. It is important to remember to always give the digits their correct value. For example:
500 take away no other hundreds is 500.
Next we have 10 take away 80. The children may tell you it can’t be done, so you introduce negative numbers. (explicit teaching opportunity), however, there may be a child who is able to tell you that it’s a negative number (or minus) 70.
Once tens column is done, it may give the child the idea about negative numbers and therefore be able to give the correct response for 3 minus 5. This gives easier numbers to work with.
500 take away 70 is 430. 430 take away is 428.
**Multiplication**

**Multiplication Vocabulary**
The symbol for multiplication is (x)

**Other words for multiplication are:** groups of, multiplicand x multiplier = product

(56 x 34= 1904)

Arrays

4x3 = 12 (rows), 3x4 = 12 (columns)  5x2 = 10 (columns), 2x5 = 10 (rows)

**MENTAL CALCULATIONS**
These are a selection of mental calculation strategies:

**Doubling and halving**
Applying the knowledge of doubles and halves to known facts.
e.g. 8 x 4 is double 4 x 4 and 12 x 5 = 6 x 10 = 3 x 20

**Using multiplication facts**
Number Facts should be developed and the connections within and between number facts understood.

Grade 2  recognise increasing and decreasing number sequences involving 2s, 4s, 3s, 5s and 10s

Grade 3  2s multiplication facts
          3s multiplication facts
          4s multiplication facts
          5s multiplication facts
          10s multiplication facts

Grade 4  Develop understanding of and recall all multiplication facts up to 10 x 10 and related division facts.

Grades 5 & 6  Develop understanding and recall efficiently all multiplication facts up to 12 x 12 and related division facts.

**Using and applying known facts**
Children should be able to utilise their number facts knowledge to derive other facts. e.g. If I know 3 x 7 = 21, what else do I know? 7 x 3 = 21; 21 / 7 = 3; 21 / 7 = 3; 1/3 of 21 = 7....

30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc.

**Use closely related facts already known**
13 x 11 = (13 x 10) + (13 x 1)
          = 130 + 13
          = 143
Multiplying by 10 or 100
Knowing that the effect of multiplying by 10 is a shift in the digits one place value position to the left so that the 'added zero' now holds a place in the ones e.g. $23 \times 10 = 230$.
Knowing that the effect of multiplying by 100 is a shift in the digits two place value position to the left so that the 'added zeros' now holds places in the ones and the tens e.g. $23 \times 100 = 2300$.

Partitioning
$23 \times 4 = (20 \times 4) + (3 \times 4)$
$= 80 + 12$
$= 92$

Compensation
$23 \times 4 = (25 \times 4) - 8$ (adding 2 to 23 to make a convenient 25 four times = 8) so $100 - 8 = 92$

Use of factors
$8 \times 12 = 8 \times 4 \times 3$

Alternative Written Methods

**Expanded Notation 1: Order is not important**

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<thead>
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<tbody>
<tr>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

When using this method, expand the two factors, so that child can see and understand the Place Value of each of the numbers.
Next to each 'answer' write the number sentence so that students can distinguish the parts of the problem completed and also sustaining their understanding of the Place Value.
Now add to obtain the product.

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<tbody>
<tr>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td>1</td>
<td>8</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>9</td>
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</tbody>
</table>

**Expanded Notation 2: Order is not important**

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<tbody>
<tr>
<td>5</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>3</td>
<td>4</td>
<td></td>
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</tbody>
</table>

a. $4 \times 6 = 24$
Write 24 in the appropriate ‘Place Value’ columns
b. $4 \times 50$ (make sure you say say “50” not “5”) is 200.
Also verbalise the fact that 4x5=20 so 4x50 is 200
Write 200 in the appropriate Place Value Columns.
c. $30 \times 6$ (make sure you say say “30” not “3”) is 180.
Write 180 in the appropriate Place Value Columns.
d. $30 \times 50$ (make sure you say say “30” not “3” etc.) is 1500.
Write 1500 in the appropriate Place Value Columns.
Now add to obtain the product. (Refer to addition if needed)
Using the LATTICE method for 56x34

1/ Write 56x34 as per grid. So it is 2 boxes (56) x 2 boxes (34).
2/ Divide each box with a diagonal line.
3/ Now work the whole grid; as per reading a map. Order does not matter.
4/ 5x3 is 15.
   Place the 1 in the top half of the box (under the 6)
5/ 6x3 is 18
   Place the 1 in the top half of the box (under the 6)
   Place the 8 in the bottom half of the box.
6/ 4x5 (or 4x5) is 20
   Place the 2 in the top half of the box (under the 15)
   Place the 0 in the bottom half of the box.
7/ 6x4 (or 4x6) is 24
   Place the 2 in the top half of the box (under the 18)
   Place the 4 in the bottom half of the box.
8/ Now add on the diagonal.
   Ones = 4
   Tens = 8 + 2 + 0. Record as 0 tens but 1 hundred is added to the hundreds diagonals.
   Hundreds = 2 + 5 + 1 + 1 (regrouped) = 9
   Thousand = 1
   So the answer is 1,904.
**TU x TU**
(Long multiplication - multiplication by more than a single digit)

72 x 38
Children will approximate first
72 x 38 is approximately 70 x 40 = 2800

```
  x  70   2  
30  2100 60  2100
  8  560  16  +  560
         +  60
         +  16
          2736
```

**ThHTU x U**
(Short multiplication - multiplication by a single digit)

4346 x 8
Children will approximate first
4346 x 8 is approximately 4346 x 10 = 43460

```
  x  4000  300  40   6  32000  2400  320  48
  8  32000 2400  320  48
         + 2400
         + 320
         + 48
          34768
```

**HTU x TU**
(Long multiplication - multiplication by more than a single digit)

372 x 24
Children will approximate first
372 x 24 is approximately 400 x 25 = 10000

```
  x  300   70   2  6000  1400  40
20  6000 1400  40
  4 1200  280   8
         + 1400
         + 1200
         + 280
         + 40
         +  8
          8928
```
Division

Division Vocabulary
The symbol for division is (÷)

Other words for division are: divide, divided by, shared between, divided into, shared among, shared equally or evenly, split up, groups of, how many groups of, equal parts, distributed among, broken up into equal parts, goes into, sharing fairly, how much each receives, how many each gets

MENTAL CALCULATIONS
These are a selection of mental calculation strategies:

Repeated Subtraction: 12 ÷ 3 is the same as 12 - 3 - 3 - 3

Multiplication and Division are inverse operations, which mean they are opposite, but they also go together. It means that if you know a multiplication fact such as 2 × 4 = 8, you also know related facts such as 4 × 2 = 8, 8 ÷ 2 = 4, 8 ÷ 4 = 2

Doubling (× 2) Halving (÷ 2)
In the number sentence 60 ÷ 10 = 6: 60 is the dividend (number being divided), 10 is the divisor (number divided by) and 6 is the quotient (answer)
Knowing that halving is dividing by 2 and that the 2 halves must be the same (equal)
Doubling is multiplying by 2

Dividing by 10 or 100
Knowing that the effect of dividing by 10 is a shift in the digits one place to the right. eg. if an item is $25 and there is a 10% discount that means 25 ÷ 10 which is 2.5 or a $2.50 discount - check it my multiplying - 10 × $2.50 = $25
Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.
(Also refer to multiplication)

Alternative Written Methods

Using a Number Line
24 ÷ 4 = 6

24 - 4, 20 - 4, 16 - 4, 12 - 4, 8 - 4, 4 - 4 - I took 4 away from 24 six times.
Short division (chunking method) $TU ÷ U$

$72 ÷ 3$ [or $3 \times ? = 72$] using inverse operation and algebra

\[
\begin{array}{c}
24 \\
3 ) 72 \\
3(10) \quad -30 \\
\phantom{3}(10) \quad \phantom{3}42 \\
3(4) \quad -12 \\
\phantom{3}(4) \quad \phantom{3}0
\end{array}
\]

Start with an appropriate estimation for how many 3s in 72 so

$10 \times 3 = 30$, now subtract 30 from the original 72.

$10$ is a 'safe' estimation for how many 3s in 42 so

$10 \times 3 = 30$, now subtract 30 from 42.

answer is $12$ and there are 4 threes in $12$.

Finally, add the numbers in the brackets.

This can be supported with images:

Moving on to using larger multiples of 10

\[
\begin{array}{c}
24 \\
3 ) 72 \\
3(20) \quad -60 \\
\phantom{3}(20) \quad \phantom{3}4 \\
3(4) \quad -12 \\
\phantom{3}(4) \quad \phantom{3}0
\end{array}
\]

\[
\begin{array}{c}
20 \\
\phantom{20}4
\end{array}
\]

\[
\begin{array}{c}
20 \\
\phantom{20}4
\end{array}
\]

\[
\begin{array}{c}
20 \\
\phantom{20}4
\end{array}
\]

Prepared by Patrizia Rinaldo with permission from Rob Vingerhoets.
Short division (chunking method) HTU ÷ U
196 ÷ 6 [or 6 × ? = 196] using inverse operation and algebra

\[
\begin{array}{c}
\text{6 ) 196} \\
- 180 \\
\hline
16 \\
- 12 \\
\hline
4
\end{array}
\]

Answer: 32 remainder 4 should be shown as 32 4/6 or 32 2/3 or 32.66

Long division HTU ÷ TU
972 ÷ 36

\[
\begin{array}{c}
\text{36 ) 972} \\
- 720 \\
\hline
252 \\
- 180 \\
\hline
72 \\
- 36 \\
\hline
0
\end{array}
\]

Answer: 27